

## The Complete Electronic multiplication in the Facel Region of a Parabability Notice

DENTE LEGIE

A CONTRACTOR OF THE PARTY OF TH

Am Force Systems commind

, Tucon PouriER Acting Chief Maroways Detection Techniques Br. Electromagnetic Sciences Division Market .

1	Unclassified SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)	
(ILL)	REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
V	RADC-TR-76-295	3. RECIPIENT'S CATALOG NUMBER
0	S. TITLE (and Sability)	5. TYPE OF REPORT & PERIOD COVERED
9	THE COMPLETE ELECTROMAGNETIC FIELDS IN THE FOCAL REGION OF A PARABOLOIDAL	In-House
	REFLECTOR	6. PERFORMING ORG. REPORT NUMBER
D	Ronald L. Fante Richard L. Taylor	S. CONTRACT OR GRANT NUMBER(s)
	Deputy for Electronic Technology (RADC/ETEP) Hanscom AFB, Massachusetts 01731	PROGRAM ELEMENT PROJECT, TASK AREA & WORK UNIT NUMBERS 6 1 102 F 2 1530201
	11. CONTROLLING OFFICE NAME AND ADDRESS Deputy for Electronic Technology (RADC/ETEP) Hanscom AFB,	September 1976
	Massachusetts 01731	19
	14. MONITORING AGENCY NAME & ADDRESS Address from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	73PP.	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
	16. DISTRIBUTION STATEMENT (of this Report)	
(16)	Approved for public release; distribution unlimite	
	17. DISTRIBUTION STATEMENT (of the abetract entered in Block 20, If different from	m Report)
	18. SUPPLEMENTARY NOTES	
	19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Antennas Reflectors Electromagnetics	
	By using the physical optics approximation we have program to calculate the complete near field distributed is illuminated by a plane wave. This programumber of results obtained with this program are comparisons with known results for the fields in the second comparisons.	ribution of a paraboloid am is described and a presented, including some

DD 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

309050

### Contents

1:	INTRODUCTION	
2.	THEORETICAL BACKGROUND	*
3.	RESULTS	
AP	PENDIX A	1

## Illustrations

1.	Reflector Geometry	6
2.	Magnitude of the Fields Along the yo = 0 Axis for z = 0.95F	10
3.	Phase of the Fields Along the y = 0 Axis for z = 0.95F	10
4.	Magnitude of the Fields Along the x = 0 Axis for z = 0.95F	10
5.	Phase of the Fields Along the x = 0 Axis for z = 0.95F	10
6.	Magnitude of the Fields Along the y = 0 Axis for z = 0.967F	12
7.	Phase of the Fields Along the y = 0 Axis for z = 0.967F	12
8.	Magnitude of the Fields Along the y = 0 Axis for z = 0.983F	12
9.	Phase of the Fields Along the y = 0 Axis for z = 0.983F	12
10.	Magnitude of the Fields Along the x <sub>o</sub> = 0 Axis for z <sub>o</sub> = F (focal plane)	13

OISTRIBUTION/AVAILABILITY COOKS

Dist. Avail. and/or SPERML

White Section
Buff Section

000

#### Illustrations

11.	Phase of the Fields Along the $x_0 = 0$ Axis for $z_0 = F$ (focal plane)	13
12.	Magnitude of the Fields Along an Axis Oriented at 45° with Respect to the x Axis, for z = 0.967F	13
13.	Geometry for the Results in Figure 12	13

The second rest

# The Complete Electromagnetic Fields in the Focal Region of a Paraboloidal Reflector

#### 1. INTRODUCTION

In designing optimum feed systems for Cassegrain reflector systems, it is highly desirable to have an accurate picture of the electromagnetic fields in the reflector focal region. In order to study these fields we have considered the case of a plane wave incident upon a large reflector, as shown in Figure 1, and have used the physical optics approximation to calculate the complete electromagnetic field distribution produced in the vicinity of the reflector focus.

#### 2. THEORETICAL BACKGROUND

Let us consider a plane wave with electric and magnetic fields\*

$$\underline{\mathbf{E}}_{\mathbf{i}} = \mathbf{E}_{\mathbf{o}} \hat{\mathbf{y}} \exp \left[ \mathbf{i} \left( \omega \mathbf{t} + \mathbf{k} \mathbf{z} \right) \right] , \qquad (1a)$$

$$\frac{H_i}{-1} = H_0 \hat{x} \exp \left[ i(\omega t + kz) \right] , \qquad (1b)$$

<sup>(</sup>Received for publication 29 September 1976)

<sup>\*</sup>In Eq. (1)  $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  are unit vectors along x, y, and z. Also k is the wavenumber =  $2\pi/\lambda$ , where  $\lambda$  is the signal wavelength.

incident from the right upon the reflector in Figure 1. If we assume that the reflector surface is described by the arbitrary function

$$z = f(x, y) , (2)$$

it can be shown that the magnetic field scattered by the reflector is given, in the physical optics approximation, by

$$\underline{H}_{S} = -\frac{H_{O}}{2\pi} \iint_{S_{O}} dxdy \left\{ \left( \hat{z} - \frac{\partial f}{\partial x} \hat{x} - \frac{\partial f}{\partial y} \hat{y} \right) \times \hat{x} e^{ikz} \right\} \times \nabla \left( \frac{e^{-ikR}}{R} \right), \quad (3)$$

where R is the distance from a source point (x, y, z) on the reflector to the field point  $(x_0, y_0, z_0)$ ,  $S_0$  is the projection of the reflector surface onto the x - y plane and x, y, and z are unit vectors.

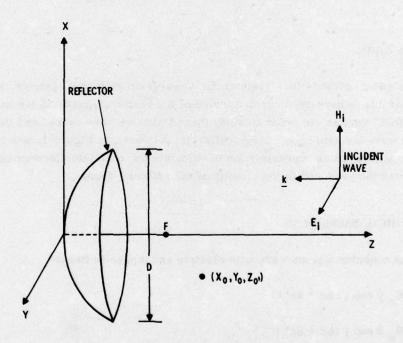


Figure 1. Reflector Geometry

<sup>1.</sup> Silver, S. (1965) Microwave Antenna Theory and Design, Dover (New York).

If we assume that the reflector surface is a parabola with a focus at z = F, then Eq. (2) becomes

$$z = \frac{1}{4F} (x^2 + y^2) . (4)$$

and the projection of the reflector onto the x-y plane is a circle satisfying the equation

$$x^2 + y^2 = \left(\frac{D}{2}\right)^2 \quad , \tag{5}$$

where D is the diameter of the reflector. If we now use Eqs. (4) and (5) in (3) we obtain, after some manipulation

$$\underline{H}_{S}(x_{o}, y_{o}, z_{o}) = \frac{H_{o}}{2\pi} \int_{-D/2}^{D/2} dx \int_{-\gamma(x)}^{\gamma(x)} dy \phi (x, y)$$

$$\times \left\{ \hat{x} \left[ z_{o} - \frac{x^{2}}{4F} + \frac{y^{2}}{4F} - \frac{y_{o}y}{2F} \right] + \hat{y} \frac{(x_{o} - x)y}{2F} - \hat{z} (x_{o} - x) \right\} , \quad (6)$$

where

$$\gamma(x) = \left[ \left( \frac{D}{2} \right)^2 - x^2 \right]^{1/2} ,$$

$$\phi(x, y) = \left( \frac{ik}{R^2} + \frac{1}{R^3} \right) \exp \left[ -ik \left\{ R - \frac{x^2}{4F} - \frac{y^2}{4F} \right\} \right] ,$$

$$R = \left[ (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right]^{1/2} .$$

The electric field distribution can be obtained by employing the Maxwell equation

$$\nabla \times \mathbf{H}_{\mathbf{S}} = \mathbf{i}\omega \,\epsilon_{\mathbf{0}} \,\mathbf{E}_{\mathbf{S}} \quad . \tag{7}$$

The result for Es is

$$E_{x}(x_{o}, y_{o}, z_{o}) = \frac{H_{o}}{2\pi i \omega \epsilon_{o}} \int_{-D/2}^{D/2} \int_{-\gamma(x)}^{\gamma(x)} dy (x_{o} - x) \left[ y_{o} - y + \frac{y}{2F} z_{o} - \frac{y}{2F} \left( \frac{x^{2}}{4F} + \frac{y^{2}}{4F} \right) \right] \theta (x, y),$$
(8a)

$$E_{y}(x_{o}, y_{o}, z_{o}) = \frac{H_{o}}{\pi i \omega \epsilon_{o}} \int_{-D/2}^{D/2} dx \int_{-\gamma(x)}^{\gamma(x)} dy \quad \phi(x, y)$$

$$-\frac{H_{o}}{2\pi i \omega \epsilon_{o}} \int_{-D/2}^{D/2} dx \int_{-\gamma(x)}^{\gamma(x)} dy \quad \theta(x, y) \left\{ \alpha(x, y) \left[ z_{o} - \frac{x^{2}}{4F} - \frac{y^{2}}{4F} \right] + (x_{o} - x)^{2} \right\},$$
(8b)

$$E_{\mathbf{z}}(\mathbf{x}_{0}, \mathbf{y}_{0}, \mathbf{z}_{0}) = \frac{H_{0}}{i \omega \epsilon_{0} \pi} \int_{-D/2}^{D/2} \int_{-\gamma(\mathbf{x})}^{\gamma(\mathbf{x})} d\mathbf{y} \frac{\mathbf{y}}{2\mathbf{F}} \phi(\mathbf{x}, \mathbf{y})$$

$$- \frac{H_{0}}{2\pi i \omega \epsilon_{0}} \int_{-D/2}^{D/2} \int_{-\gamma(\mathbf{x})}^{\gamma(\mathbf{x})} d\mathbf{y} \theta(\mathbf{x}, \mathbf{y}) \left\{ \frac{\mathbf{y}}{2\mathbf{F}} (\mathbf{x}_{0} - \mathbf{x})^{2} - (\mathbf{y}_{0} - \mathbf{y}) \alpha(\mathbf{x}, \mathbf{y}) \right\} , \quad (8c)$$

where

$$\alpha(x, y) = z_0 - \frac{x^2}{4F} + \frac{y^2}{4F} - \frac{y_0 y}{2F}$$
,

$$\theta (x,y) = \left\{ \frac{3}{R^5} + \frac{i3k}{R^4} - \frac{k^2}{R^3} \right\} \exp \left[ -ik \left( R - \frac{x^2}{4F} - \frac{y^2}{4F} \right) \right].$$

Equations (6) and (8) are the formal expressions for the complete electromagnetic fields in the physical optics approximation. They represent a quite good approximation for the entire region  $z > \frac{1}{4F}(x^2 + y^2)$ , which is of interest to us. Of course, they are inaccurate for z < 0; over part of that region the geometrical theory of diffraction must be employed.

By observing Eqs. (6) and (8) it is clear that the scattered electric and magnetic fields possess certain symmetry properties. These are (for a fixed z<sub>o</sub>)

$$H_{x}(x_{o}, -y_{o}) \approx H_{x}(x_{o}, y_{o})$$
, (9a)

$$H_{x}(-x_{o}, y_{o}) = H_{x}(x_{o}, y_{o})$$
 (9b)

$$H_{v}(x_{0}, -y_{0}) = -H_{v}(x_{0}, y_{0})$$
, (9c)

$$H_{v}(-x_{o}, y_{o}) = -H_{v}(x_{o}, y_{o})$$
, (9d)

$$H_{z}(x_{o}, -y_{o}) = H_{z}(x_{o}, y_{o})$$
 (9e)

$$H_z(-x_0, y_0) = -H_z(x_0, y_0)$$
 (9f)

$$E_{x}(x_{o}, -y_{o}) = -E_{x}(x_{o}, y_{o})$$
 (10a)

$$E_{x}(-x_{o}, y_{o}) = -E_{x}(x_{o}, y_{o})$$
, (10b)

$$E_{y}(x_{o}, -y_{o}) = E_{y}(x_{o}, y_{o})$$
 (10c)

$$E_{y}(-x_{o}, y_{o}) = E_{y}(x_{o}, y_{o})$$
 (10d)

$$E_z(x_0, -y_0) = -E_z(x_0, y_0)$$
 (10e)

$$E_z(-x_0, y_0) = E_z(x_0, y_0)$$
 (10f)

Because of the aforementioned symmetry properties we have calculated  $\underline{E}$  and  $\underline{H}$  only for positive values of  $x_0$  and  $y_0$ ; the values for negative  $x_0$ ,  $y_0$  follow immediately from Eqs. (9) and (10).

#### 3. RESULTS

We have developed a computer program to calculate the field components given by Eqs. (6) and (8). As an example of typical results of our program, we have studied a reflector such that

$$\frac{\mathbf{F}}{\mathbf{D}} = \frac{1}{3} \quad , \tag{11a}$$

$$\frac{D}{\lambda} = 60 , \qquad (11b)$$

where  $\lambda$  is the signal wavelength, and have calculated the field distribution in the planes  $z_0 = 0.95F$ , 0.967F, 0.983F, and 1.0F. In Figures 2 and 3 we show\* the amplitude and phase of the electric and magnetic fields in the plane  $z_0 = 0.95F$ . The fields shown are those along the line  $y_0 = 0$ , for differing values of  $x_0$ . In Figures 4 and 5 we show the fields along the line  $x_0 = 0$  for differing values of  $y_0$ .

<sup>\*</sup>In all the results of Figures 2 to 11 we have assumed  $H_0 = 2\pi$ .

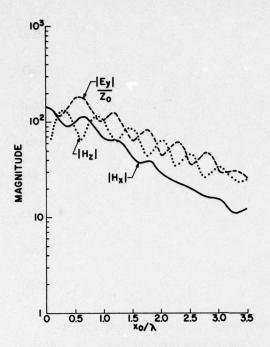


Figure 2. Magnitude of the Fields Along the  $y_0$  = 0 Axis for z = 0.95F

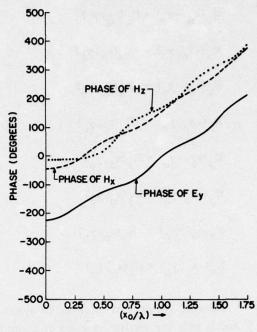


Figure 3. Phase of the Fields Along the  $y_0 = 0$  Axis for  $z_0 = 0.95F$ 

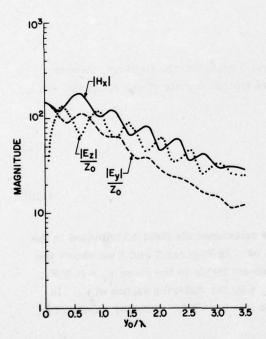


Figure 4. Magnitude of the Fields Along the  $x_0 = 0$  Axis for  $z_0 = 0.95F$ 

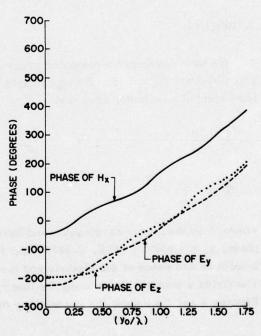


Figure 5. Phase of the Fields Along the  $x_0 = 0$  Axis for  $z_0 = 0.95F$ 

We note, upon comparing Figures 2 and 4, that if the results are known along the  $y_0 = 0$  axis we can immediately obtain those along the axis  $x_0 = 0$  by replacing  $|H_x|$  by  $|E_y|/Z_0$ ,  $|H_z|$  by  $|E_z|/Z_0$ , and  $|E_y|/Z_0$  by  $|H_x|$ , where  $Z_0$  is the impedance of vacuum. Because of this duality, in the remaining figures we will only show results along either the  $x_0 = 0$  or the  $y_0 = 0$  axis. In Figures 6 and 7 we show the fields in the plane  $z_0 = 0.967F$ ; in Figures 8 and 9 we show the results in the plane  $z_0 = 0.983F$ , and finally, in Figures 10 and 11 we show the results in the focal plane. The components  $E_x$  and  $H_y$  are now shown because they are both zero.

There are several observations which should be made regarding our results:

(1) The results of Figure 10 for the transverse fields in the focal plane agree with those calculated earlier by Minnett et al.  $^2$  (see Figure 11 of Minett's paper, for our case  $^0$ 0~  $^{74^\circ}$ ).

(2) The cross-polarized fields  $|E_z|/Z_0$  and  $|H_z|$  are generally of the same order as  $|H_x|$  and  $|E_y|/Z_0$ , except very near to the  $z_0$  axis. This is true even in the focal plane, and even holds within the focal spot (that is, we call the transverse dimension of the first null in Figure 10 the focal spot size, and this is of order  $\lambda F/D$ ) as can be seen from Figure 10, where  $|E_z|/Z_0$  is small near the center of the focal spot ( $y_0$  near zero) but is large near the outer edge ( $y_0 \approx 0.5\lambda$ ).

(3) In Figures 2 to 11 we have shown the fields on the x and y axes where  $E_x = H_y = 0$ . This should not imply that  $E_x = H_y = 0$  off these axes. In Figure 12 we show the field distribution along an axis (see Figure 13) oriented 45° relative to the x axis in the plane  $z_0$ . Note that both  $E_x$  and  $H_y$  are nonzero, although they are considerably smaller than the other field components.

Minnett, H. C. and Thomas, B. (1968) Fields in the image plane of symmetrical focusing reflectors. Proc. IEEE, 115:1419-1430.

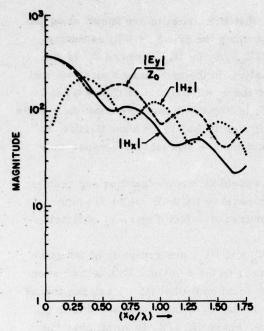
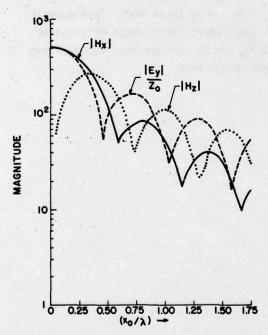


Figure 6. Magnitude of the Fields Along the  $y_0 = 0$  Axis for  $z_0 = 0.967$ F

Figure 7. Phase of the Fields Along the  $y_0 = 0$  Axis for  $z_0 = 0.967$ F



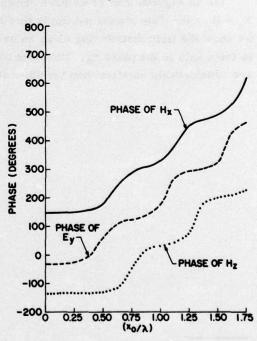


Figure 8. Magnitude of the Fields Along the  $y_0 = 0$  Axis for  $z_0 = 0.983F$ 

Figure 9. Phase of the Fields Along the  $y_0 = 0$  Axis for  $z_0 = 0.983$ F

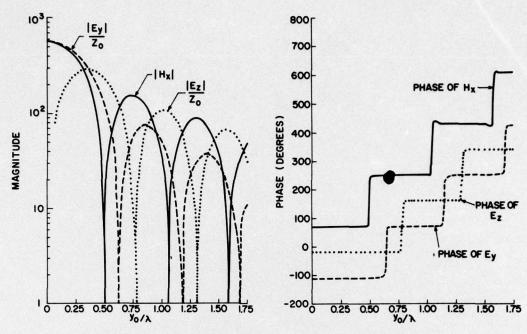


Figure 10. Magnitude of the Fields Along the  $x_0 = 0$  Axis for  $z_0 = F$  (focal plane) Figure 11. Phase of the Fields Along the  $x_0 = 0$  Axis for  $z_0 = F$  (focal plane)

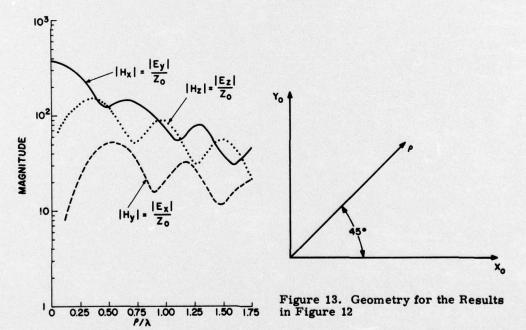


Figure 12. Magnitude of the Fields Along an Axis Oriented at  $45^{\rm O}$  with Respect to the x Axis, for  $z_{\rm O}$  = 0.967F

### Appendix A

In this appendix we present a Fortran listing of the computer program used to calculate  $\underline{E}$  and  $\underline{H}$ . Note that the quantities printed out for  $\underline{E}$  are actually the electric field normalized by  $Z_0$  rather than  $\underline{E}$ . The inputs to the program are:

D = diameter of reflector,

F = focal length,

 $K = wavenumber = 2\pi/\lambda$ ,

X0, Y0, Z0 = coordinates of observation point,

XTOL = YTOL usually set to  $10^{-3}$ .

```
SUBROUTINE XINTEG(XB, XA) INPLICIT COMPLEX(Q)
   CCMMON/QXI/QXINTEG (6)
   COPMON/QYI/QYINTEG(6)
   COMMON/XTOL/TOL
   DIMENSION QTWO(6), QFOUR(6), QENDS(6), QTOTAL(6)
   H = (XB - XA) / 2.0
N = 1
CALL YINTEG(H+XA)
DO 10 J=1,6
   H = (XB - XA) / 2.0
DO 10 J=1,6
QTHC(J) = (0.0, 0.0)
10 QFCUR(J) = QYINTEG(J)
CALL YINTEG(XA)
   DO 10 J=1,6
   DO 20 J=1,6
20 QENDS(J) = QYINTEG(J)
   CALL YINTEG(XB)
   00 30 J=1,6
   QENDS(J) = QENDS(J) + QYINTEG(J)
30 QTCTAL(J) = (QENDS(J) + 4.0*QFQUR(J) ) * H / 3.0
50 QXINTEG(J) = QTOTAL(J)
   Y = H = H / 2.0
   N = 2 * N
   DO 60 J=1,6
   QTWO(J) = QTWO(J) + QFOUR(J)
60 QFOUR(J) = (0.0, 0.0)
   I = 0
70 I = I + 1
CALL YINTEG (Y+XA)
   DO 80 J=1,6
80 QFCUR(J) = QFOUR(J) + QYINTEG(J)
   Y = Y + H + H
   IF(I .LT. N) GO TO 70
   IFLAG = 0
   PRINT *
   DO 90 J=1,6
   QTOTAL(J) = (QENDS(J) + 2.0 GTHO(J) + 4.0 GFOUR(J)) + H / 3.0
   QDENOM = QTOTAL(J)
   IF (CAES (QDENOM) .LT. TOL) QDENOM = CMPLX(TOL, 0.0)
   IF (CABS ( (GXINTEG (J) - QTCTAL (J) )/QDENOM) .GT. TOL) IFLAG = 1
   PRINT *, CABS( QTOTAL(J) )
90 CONTINUE
   IF (IFLAG .EQ. 1) GO TO 40
   DO 100 J=1,6
100 QXINTEG(J) = QTOTAL(J)
   RETURN
   END
```

```
SUBROLITINE YINTEG(X)
   IMPLICIT COMPLEX(Q)
   COMMON/ONE/XO, ONEOSF, ONEOSF, ZO, K, YO, DOYSSQ, OCONST
   COMMON/QQQ/QINT(6)
   COMMON/QYI/QYINTEG(6)
   COMMON/XXX/XDIFF, XDIFF2, XSQ, EX
   COMMON/YTOL/TOL
   DIMENSION QTHO(6), GFOUR(6), GENDS(6), QTOTAL(6)
   EX = X
   XSQ = X**2
   XDIFF2 = XOIFF**2
   XOIFF = X0 - X
   YB = GAMX = SQRT(DOV2SQ - XSQ)
   YA = -GAMX
   H = GAMX
   N = 1
   CALL FIELDS (H+YA)
   DO 10 J=1,6
QTWO(J) = (0.0, 0.0)
10 QFCUR(J) = QINT(J)
   CALL FIELDS (YA)
   00 20 J=1,6
20 QENDS(J) = QINT(J)
   CALL FIELDS (YB)
   00 30 J=1,6
   QENDS(J) = QENDS(J) + QINT(J)
30 QTOTAL(J) = (QENDS(J) + 4.0*QFCUR(J) ) * H / 3.0
40 DO 50 J=1,6
 50 QYINTEG(J) = QTOTAL(J)
   Y = H = H / 2.0
N = 2 + N
   DO 60 J=1,6

OTMO(J) = OTMO(J) + OFOUR(J)
60 QFOUR(J) = (0.0, 0.0)
   I = 0
70 I = I + 1
   CALL FIELDS (Y+YA)
   00 80 J=1,6
 80 OFOUR(J) = QFOUR(J) + GINT(J)
   Y = Y + H + H
   IF(I .LT. N) GO TO 70
   IFLAG = 0
   DO 90 J=1,6
   QTOTAL(J) = (QENOS(J) + 2.0*QTNO(J) + 4.0*QFOUR(J) ) * H / 3.0
   QDENOM = QTOTAL(J)
   IF (CABS (QDENOM) .LT. TOL) QDENOM = CMPLX(TOL, 0.0)
   IFICABSE (GYINTEGEL) - QTOTAL(J) )/QUENON) .GT. TOL) IFLAG = 1
 90 CONTINUE
   IF(IFLAG . EQ. 1) GO TO 40
   00 100 J=1,6
100 QYINTEG(J) = QTOTAL(J)
   RETURN
   END
```

```
SUBROUTINE FIELDS(Y)
IMPLICIT COMPLEX(Q)
COMMON/ONE/XO, ONEOZF, ONEO4F, ZO, K, YO, DOVZSQ, GCONST
COMMON/QQQ/QINT(6)
COMMON/XXX/XDIFF, XDIFF2, XSQ, X
REAL K
YOIFF = YO - Y
YDIFF2 = YDIFF**2
TERM1 = ZO - ONEO4F*(XSQ + YSQ)
RSO = XDIFF2 + YDIFF2 + TERM1**2
R = SQRT(RSQ)
ONECR2 = 1.0 / RSQ
ONEOR = 1.0 / R
QCEXP = CEXP(CMPLX(0.0, -K*(R + TERM1 - Z0) ) )
QPHI = QCEXP + CHPLX(ONEOR, K) + ONEOR2
QTHETA = QCEXP + CHPLX(3.0*ONECR2-K**2, 3.0*K*ONEOR)*CNEOR2*ONEOR
TERM2 = 20 - ONEO4F + (XSQ - Y*(Y - 2.0*Y0) )
QINT(1) = QPHI + TERM2
QINT(3) = -QPHI + XDIFF
QINT(2) = -QINT(3) * Y * CNEO2F
QINT(4) = XDIFF * (YDIFF + ONEO2F*Y*TERM1) * QTHETA * QCONST
QINT(5) = (2.0*QPHI - QTHETA*(TERM2*TERM1 + XDIFF2) )*QCONST
QINT(6) = (2.0*QPHI*Y*CNEC2F - QTHETA*(XDIFF2*Y*ONEO2F - YDIFF*
      TERM2) ) *QCONST
RETURN
END
```

# ADESCON O

the first of the f